MAT 1341 C Midterm Exam II

March 25, 2019 Length: 80 minutes.
Professor: Rachid Bentoumi

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2	
3	
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5	
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Total	

Family name:______________________

First name:________________________

Student number:____________________

PLEASE READ THESE INSTRUCTIONS CAREFULLY:

1. You have 80 minutes to write this exam.
2. You are not allowed to consult your notes or any books. Calculators, phones, and other electronic devices are not allowed.
3. Carefully read each question and **answer all questions in the space provided for this purpose. Mark your answers to multiple choice questions in the boxes above** For questions 4 to 6, you can use the back of the pages if necessary, but do not forget to indicate it to the T.A!
4. Questions 1 to 3 are multiple choice questions, each worth 2 points. No partial credit will be awarded. You must indicate the method you used to select the correct answer; unjustified answers will not be given credit.
5. Questions 4 to 6 are long-answer questions. They are each worth 6 points. **You must justify and write your answers correctly to get all possible points.**
6. Good luck! Bonne chance!

1. Suppose that U is a subspace of $\mathbb{R}^4$ with basis $\{(1,-2,3,4),\ (-3,6,-5,-16)\}$. What is the dimension of $U^{\perp}$?

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5

2. Suppose that

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}.$$

What is the second row of A^{-1}?

A. A is not invertible
B. $(-2, -1, 1)$
C. $(-2, -2, 1)$
D. $(0, 1, 0)$
E. $(5, 4, -2)$

3. Consider the vectors

$$v_1 = (1, 2, 3), \quad v_2 = (2, 6, 8), \quad v_3 = (4, 1, 5), \quad v_4 = (-2, 5, 3),$$

each in $\mathbb{R}^3$. Which subset of these vectors forms a basis of $U = \text{span}\{v_1, v_2, v_3, v_4\}$?

A. $\{v_1\}$
B. $\{v_1, v_2\}$
C. $\{v_2, v_3, v_4\}$
D. $\{v_1, v_2, v_3\}$
E. $\{v_1, v_3, v_4\}$
F. $\{v_1, v_2, v_3, v_4\}$

4. Consider the following (3×4) matrix A:

$$A = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 0 & 2 & 2 \\ 1 & -1 & 4 & 3 \end{bmatrix}.$$

a) Find a basis of the column space $\mathrm{Col}(A)$ of A.

b) Give a **complete** geometric description of $\mathrm{Col}(A)$.

c) Find a basis of the kernel $\mathrm{Ker}(A)$ of A.

d) Calculate $\dim(\mathrm{Ker}(A)) + \dim(\mathrm{Col}(A))$.

5. Consider the subspace W of $\mathbb{R}^4$ with the basis $\{(1,0,1,0),\ (0,1,0,1),\ (0,0,0,1)\}$.

a) Find an orthogonal basis B of W.

b) Find the best approximation in W of the vector $v = (0,1,-1,1)$.

c) Extend the orthogonal basis B from part (a) to a basis of $\mathbb{R}^4$.

6(a). In each case, indicate in the corresponding box if the statement below is (always) true or can be false.

- If you think that the statement may be wrong, give an explicit example that it is false.
- If you think that the statement is (always) true, you must justify it with a clear explanation.

I. If A is an invertible matrix and $AB = 0$, then $B = 0$.

ANSWER $\boxed{}$

II. The columns of a (4×3) matrix are always linearly dependent (L.D).

ANSWER $\boxed{}$

6(b). Let A be an $(n \times n)$ matrix with real entries. Give three equivalent statements to the statement

"A is an invertible matrix"

following these specifications:

(i) One which gives a condition on the **rank of** A

(ii) One gives a condition on the **reduced row echelon form of** A

(iii) One which gives a condition on the **columns of** A